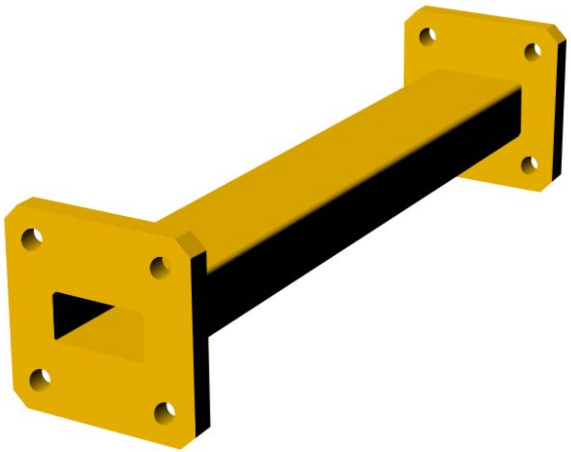
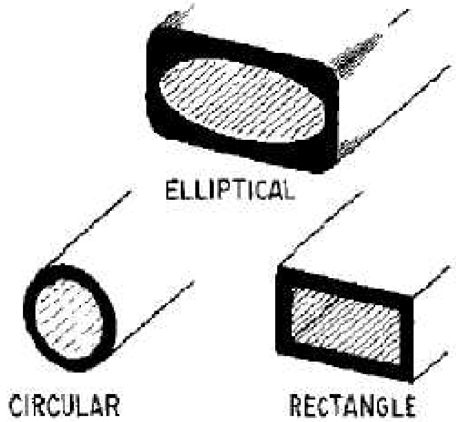
Wave guides

Wave guide field equations

Rectangular wave guides

Dielectric wave guides





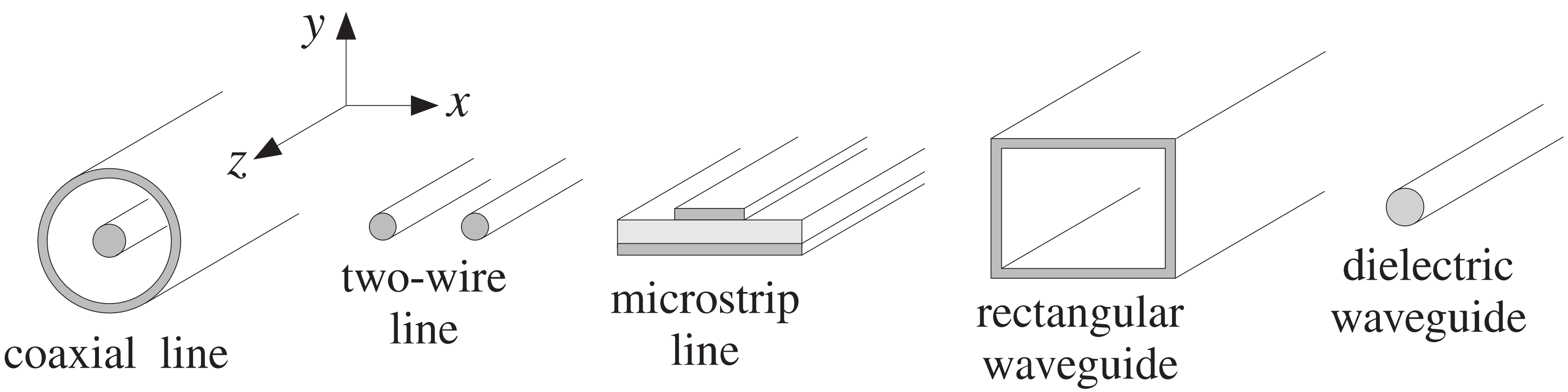


Fig. 9.0.1 Typical waveguiding structures.

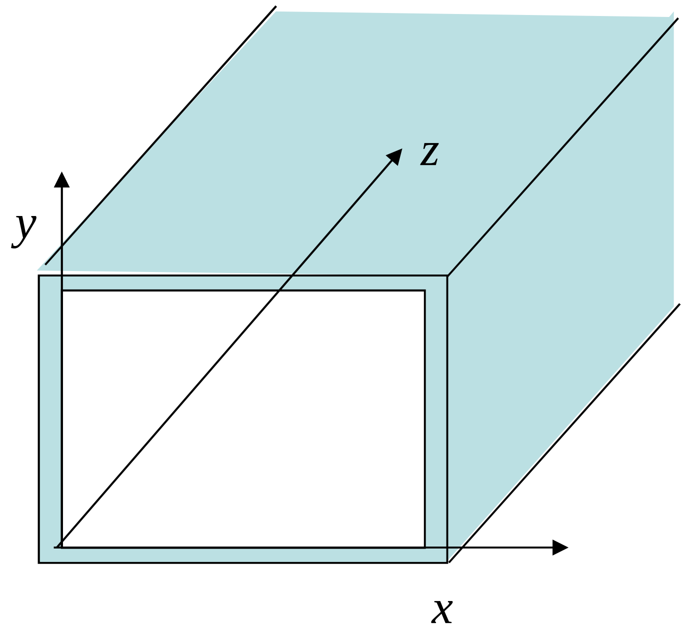
Perfect conductor wave guide

The transmission line theory only models the TEM waves. Non-TEM waves propagation requires wave guide theory.

Boundary conditions

$$\mathbf{E}_{\parallel} = 0$$

$$\mathbf{B}_{\perp} = 0$$



Here we consider the monochromatic waves propagating in the z direction

$$\mathbf{E}(x,y,z,t) = \mathbf{E}_0(x,y)\,e^{i(kz-\omega t)}$$

$$\mathbf{B}(x,y,z,t) = \mathbf{B}_0(x,y)\,e^{i(kz-\omega t)}$$

Maxwell’ s equations

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = -\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

Assume

$$\mathbf{E}_0\left(x, y\right) = E_x\hat{\mathbf{x}} + E_y\mathbf{y} + E_z\hat{\mathbf{z}}$$

$$\mathbf{B}_0\left(x, y\right) = B_x\hat{\mathbf{x}} + B_y\mathbf{y} + B_z\hat{\mathbf{z}}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{E} = \left(\frac{\partial E_z}{\partial y} - ikE_y\right)\hat{\mathbf{x}} + \left(-\frac{\partial E_z}{\partial x} + ikE_x\right)\mathbf{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$= i\omega\left(B_x\hat{\mathbf{x}} + B_y\mathbf{y} + B_z\hat{\mathbf{z}}\right)$$

$$\nabla \times \mathbf{B} = -\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{B} = \left(\frac{\partial B_z}{\partial y} - ikB_y\right)\hat{\mathbf{x}} + \left(-\frac{\partial B_z}{\partial x} + ikB_x\right)\mathbf{y} + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$= -i\frac{\omega}{c^2}\left(E_x\hat{\mathbf{x}} + E_y\mathbf{y} + E_z\hat{\mathbf{z}}\right)$$

$$\frac{\partial E_z}{\partial y} - ikE_y = i\omega B_x$$

$$-\frac{\partial E_z}{\partial x} + ikE_x = i\omega B_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z$$

$$\frac{\partial B_z}{\partial y} - ikB_y = -\frac{i\omega}{c^2} E_x$$

$$-\frac{\partial B_z}{\partial x} + ikB_x = -\frac{i\omega}{c^2} E_y$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -\frac{i\omega}{c^2} E_z$$

$$E_x = \frac{i}{\left(\omega^2/c^2\right) - k^2} \left(k \frac{\partial E_z}{\partial x} + \omega \frac{\partial B_z}{\partial y} \right)$$

$$E_y = \frac{i}{\left(\omega^2/c^2\right) - k^2} \left(k \frac{\partial E_z}{\partial y} - \omega \frac{\partial B_z}{\partial x} \right)$$

$$B_y = \frac{i}{\left(\omega^2/c^2\right) - k^2} \left(k \frac{\partial B_z}{\partial y} + \frac{\omega}{c^2} \frac{\partial E_z}{\partial x} \right)$$

$$B_x = \frac{i}{\left(\omega^2/c^2\right) - k^2} \left(k \frac{\partial B_z}{\partial x} - \frac{\omega}{c^2} \frac{\partial E_z}{\partial y} \right)$$

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + ikE_z = 0 \qquad \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + ikB_z = 0$$

$$\frac{\partial E_x}{\partial x} = \frac{i}{\left(\omega^2/c^2\right) - k^2}\left(k\frac{\partial^2 E_z}{\partial x^2} + \omega\frac{\partial^2 B_z}{\partial x \partial y}\right) \qquad \frac{\partial B_x}{\partial x} = \frac{i}{\left(\omega^2/c^2\right) - k^2}\left(k\frac{\partial^2 B_z}{\partial x^2} - \frac{\omega}{c^2}\frac{\partial^2 E_z}{\partial x \partial y}\right)$$

$$\frac{\partial E_y}{\partial y} = \frac{i}{\left(\omega^2/c^2\right) - k^2}\left(k\frac{\partial^2 E_z}{\partial y^2} - \omega\frac{\partial^2 B_z}{\partial y \partial x}\right) \qquad \frac{\partial B_y}{\partial y} = \frac{i}{\left(\omega^2/c^2\right) - k^2}\left(k\frac{\partial^2 B_z}{\partial y^2} + \frac{\omega}{c^2}\frac{\partial^2 E_z}{\partial y \partial x}\right)$$

$$\Rightarrow \quad \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \left(\frac{\omega^2}{c^2} - k^2\right)E_z = 0 \qquad \frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} + \left(\frac{\omega^2}{c^2} - k^2\right)B_z = 0$$

TE vs TM waves

TE (transverse electric) waves $E_z = 0$

TM (transverse magnetic) waves $B_z = 0$

TEM waves $E_z = B_z = 0$ Cannot exists in a hollow pipe

$$E_z = 0 \qquad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = -ikE_z = 0 \qquad B_z = 0 \qquad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z = 0$$

⇒ define $E_x = -\dfrac{\partial \phi}{\partial x}$, $E_y = -\dfrac{\partial \phi}{\partial y}$ One has $\dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} = 0$

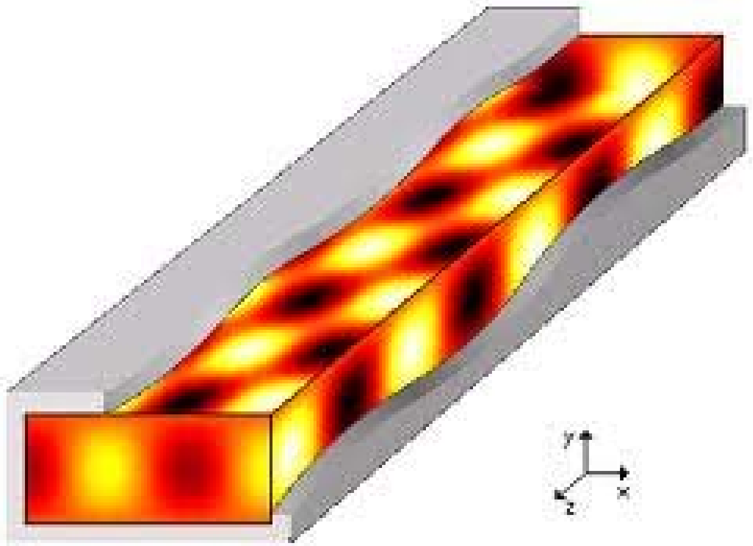
There will be no local maxima or minima inside the pipe

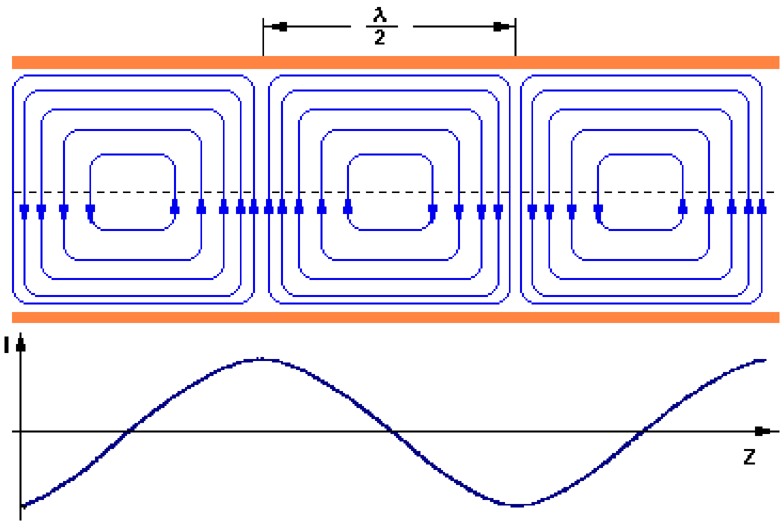
For a hollow pipe, the waveguide surface is equi-potential, so the electric potential inside the pipe will be constant, resulting in a zero field

Wave guide field equations

Rectangular wave guides

Dielectric wave guides





TE modes

Suppose a wave guide has a width a and a height b

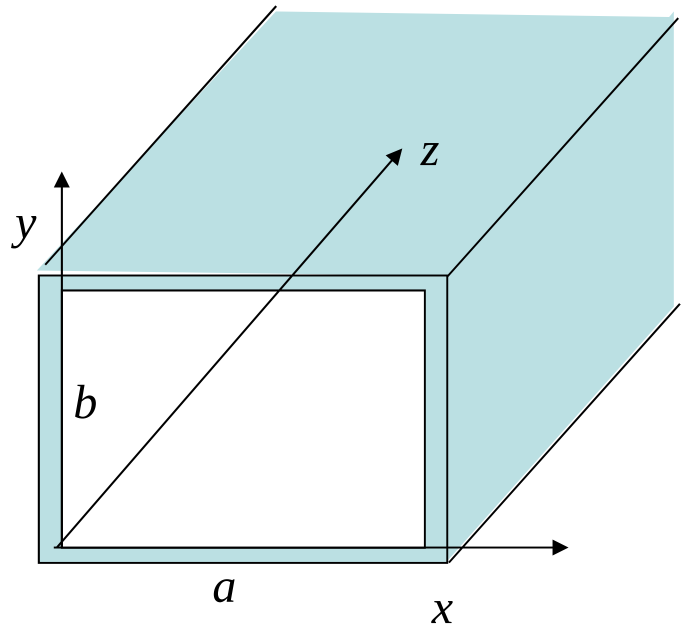
We now consider the TE waves, so only B_z is important $\qquad E_z = 0$

$$\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} + \left(\frac{\omega^2}{c^2} - k^2\right) B_z = 0$$

Use separation of variables

$$B_z = X(x)Y(y)$$

$$\Rightarrow \quad \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{\omega^2}{c^2} - k^2 = 0$$



Boundary conditions

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k_x^2$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k_y^2$$

$$-k_x^2 - k_y^2 + \frac{\omega^2}{c^2} - k^2 = 0$$

The solutions are

$$X = A\sin(k_x x) + B\cos(k_x x)$$

$$Y = C\sin(k_y y) + D\cos(k_y y)$$

At boundary ($x=0$, $x=a$)

$$\mathbf{E}_{\parallel} = 0$$

$$\mathbf{B}_{\perp} = 0$$

$$E_y = \frac{i}{\left(\omega^2/c^2\right) - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$

$$E_y = 0 \quad \Rightarrow \quad \frac{\partial B_z}{\partial x} = 0 \qquad \frac{\partial X}{\partial x} = 0$$

$$X = B\cos(k_x x)$$

$$k_x = \frac{m\pi}{a}$$

Boundary conditions

At boundary ($y=0, y=b$)

$$Y = D\cos\left(k_y y\right)$$

$$E_x = 0 \quad \Rightarrow \quad \frac{\partial B_z}{\partial y} = 0$$

$$k_y = \frac{n\pi}{a}$$

$$B_z = B_0 \cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)$$

$$k^2 = \frac{\omega^2}{c^2} - \pi^2\left(\frac{m}{a}\right)^2 - \pi^2\left(\frac{n}{b}\right)^2$$

If $\omega < \pi c\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$ $\Rightarrow$ k becomes imaginary and the wave attenuates

Cutoff frequency

$$\omega_{mn} = \pi c \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$ is called the cut-off frequency

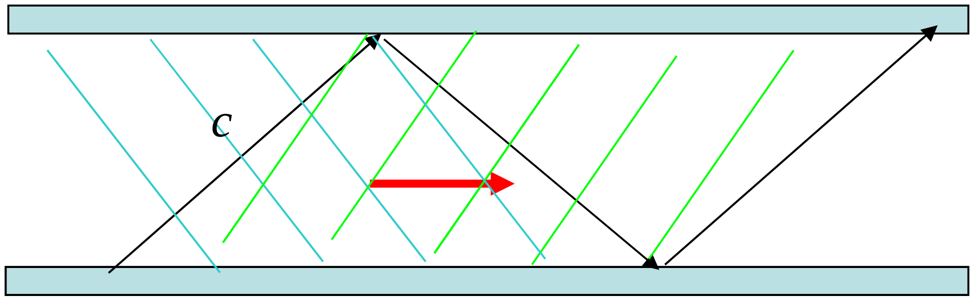
lowest cut-off frequency $\omega_{10} = \frac{\pi c}{a}$

$$k = \frac{1}{c}\sqrt{\omega^2 - \omega_{nm}^2}$$

The wave velocity $v = \frac{\omega}{k} = \frac{c}{\sqrt{1-\omega_{nm}^2/\omega^2}}$

The group velocity $v_g = \frac{d\omega}{dk} = c\sqrt{1-\omega_{nm}^2/\omega^2}$

Wave propagation



$$\mathbf{k} = (k_x, k_y, k) = \left(\frac{m\pi}{a}, \frac{n\pi}{b}, k \right)$$

The propagation angle $\cos\theta = \frac{k}{|\mathbf{k}|} = \sqrt{1 - \omega_{nm}^2 / \omega^2}$

The group velocity is the superposed wave, namely indicated by the red arrow

$$v_g = c\cos\theta = c\sqrt{1 - \omega_{nm}^2 / \omega^2}$$

The phase velocity is the speed of the wavefront,

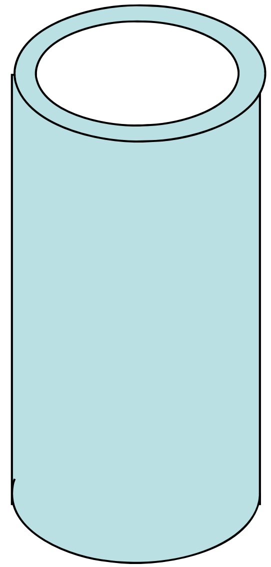
$$v = \frac{c}{\cos\theta} = \frac{c}{\sqrt{1 - \omega_{nm}^2 / \omega^2}}$$

Cylindrical waveguides

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}-\left(\frac{\omega^2}{c^2}-k^2\right)E_z=0$$

In cylindrical coordinate

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\right]E_z+\left(\frac{\omega^2}{c^2}-k^2\right)E_z=0$$



We may assume

$$E_z=R(\rho)e^{im\phi} \qquad \alpha^2=\frac{\omega^2}{c^2}-k^2$$

$$\Longrightarrow \quad \rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)R(\rho)+\left(\alpha^2\rho^2-m^2\right)R(\rho)=0$$

The Bessel function $\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)J_m(\rho)+\left(\rho^2-m^2\right)J_m(\rho)=0$

$$\Longrightarrow \quad E_z=J_m(\alpha\rho)e^{im\phi}$$

TE (transverse electric) waves $E_z = 0$

$$B_z = J_m(\alpha\rho)e^{im\phi}$$

$$k^2 = \frac{\omega^2}{c^2} - \alpha^2$$

TM (transverse magnetic) waves $B_z = 0$

$$E_z = J_m(\alpha\rho)e^{im\phi}$$

At boundary (ρ=a) $\mathbf{E}_{\|} = 0$ $E_\phi = 0$

$\mathbf{B}_\perp = 0$ $B_\rho = 0$

TE TM

$$E_\phi = \frac{i}{\left(\omega^2/c^2\right)-k^2}\left(\frac{k}{\rho}\frac{\partial E_z}{\partial \phi} + \omega\frac{\partial B_z}{\partial \rho}\right) \qquad \left.\frac{\partial B_z}{\partial \rho}\right|_a = 0 \qquad \left.\frac{\partial E_z}{\partial \phi}\right|_a = 0$$

$$B_\rho = \frac{i}{\left(\omega^2/c^2\right)-k^2}\left(k\frac{\partial B_z}{\partial \rho} + \frac{\omega}{c^2\rho}\frac{\partial E_z}{\partial \phi}\right) \qquad \left.\frac{dJ_m(\alpha\rho)}{d\rho}\right|_a = 0 \qquad J_m(\alpha a) = 0$$

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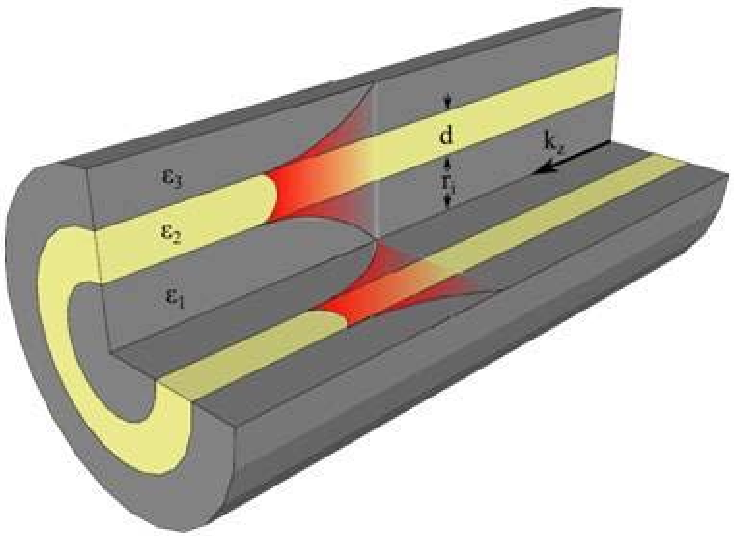
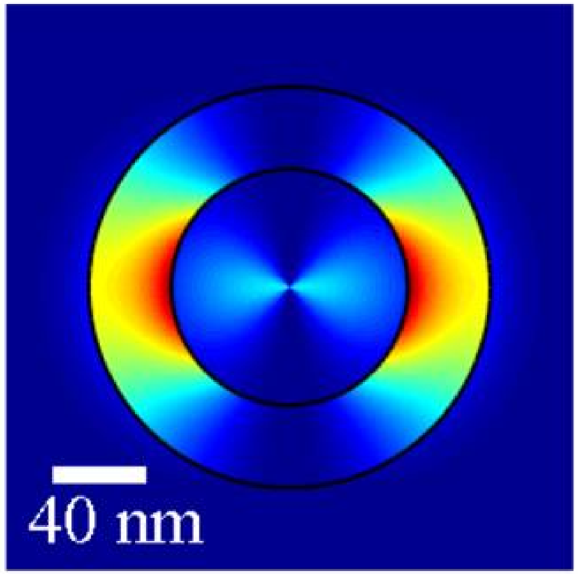


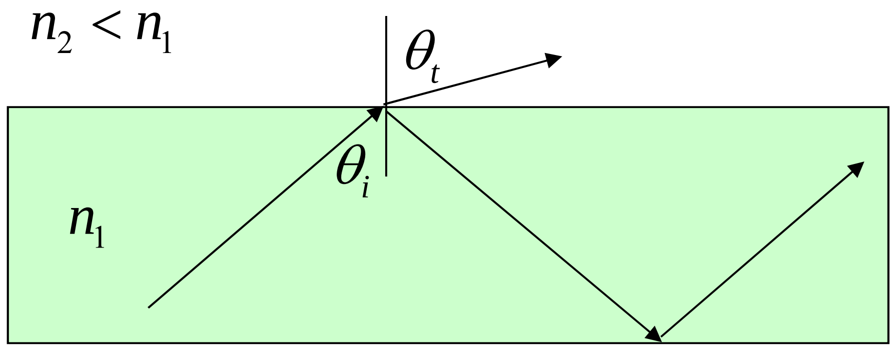


Slab of dielectric

Consider EM wave propagates along a slab of dielectric with refraction index of n_1 $\quad n = c\sqrt{\varepsilon\mu}$

On the interface, the Snell's law yields

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$



In transverse propagation

$$k_{1t} = n_1 \frac{\omega}{c}\cos\theta_i$$

$$k_{2t} = n_2 \frac{\omega}{c}\cos\theta_t$$

$$\Rightarrow$$

$$k_{1t}^2 - k_{2t}^2 = \left(n_1^2 - n_2^2\right)\left(\frac{\omega}{c}\right)^2$$

$$= k_1^2 - k_2^2 > 0$$

Total reflection

If $\sin\theta_i > \dfrac{n_2}{n_1}$ $\quad k_{1t} < \sqrt{n_1^2 - n_2^2}\,\dfrac{\omega}{c}$

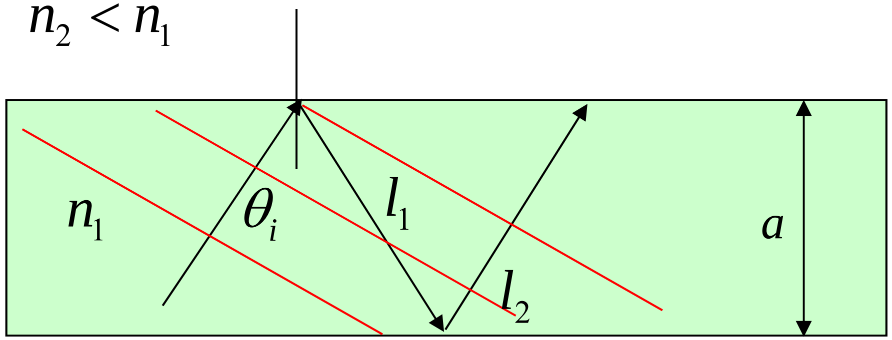
$\Rightarrow$ $k_{2t}^2 < 0$ $\quad k_{2t} = i\kappa_{2t}$ $\quad$ Evanescence wave

With this condition, the waves are reflected so the propagation along the slab is possible

In the conductor waveguide, constructive interference of incident and reflected waves is required for steady propagation. Here we need to consider the phase shift of reflection of waves on the dielectric interfaces.

Phase shift on reflection

The constructive interference requires



$$\frac{\omega}{v_1}\left(l_1 + l_2\right) + 2\phi = 2m\pi$$

$$l_2 = l_1 \cos 2\theta_i$$

$$l_1 = \frac{a}{\cos\theta_i}$$

$$\frac{\omega}{v_1}\left(1 + \cos 2\theta_i\right)\frac{a}{\cos\theta_i} + 2\phi = 2m\pi$$

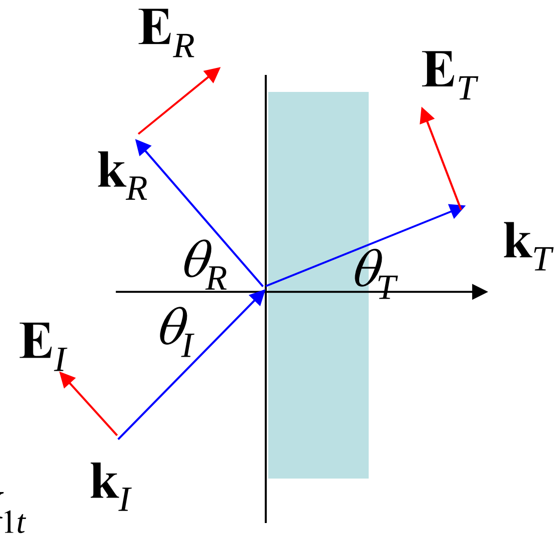
$$a\frac{\omega}{v_1}\cos\theta_i + \phi = m\pi$$

$$ak_{1t} + \phi = m\pi$$

The propagating mode equation

TM mode

The reflection coefficient for a TM mode



$$\Gamma = \frac{\alpha - \beta}{\alpha + \beta}$$

For nonmagnetic materials

$$\frac{\alpha - \beta}{\alpha + \beta} \simeq \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} = \frac{i\left(k_1/k_2\right)\kappa_{2t} - \left(k_2/k_1\right)k_{1t}}{i\left(k_1/k_2\right)\kappa_{2t} + \left(k_2/k_1\right)k_{1t}}$$

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$$

$$\alpha = \frac{\cos\theta_T}{\cos\theta_I}$$

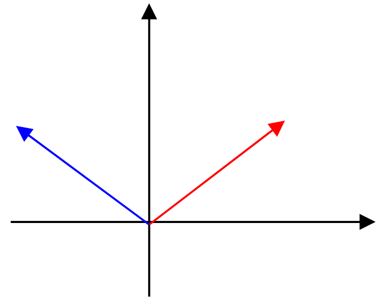
$$k_{2t} = n_2 \frac{\omega}{c} \cos\theta_t \qquad k_{1t}^2 - k_{2t}^2 = k_1^2 - k_2^2$$

$$\Rightarrow \quad \cos^2\theta_t = 1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i$$

$$\Gamma \simeq \frac{i n_1 \sqrt{\left(n_1/n_2\right)^2 \sin^2\theta_i - 1} - n_2 \cos\theta_i}{i n_1 \sqrt{\left(n_1/n_2\right)^2 \sin^2\theta_i - 1} + n_2 \cos\theta_i}$$

Here we assumed $k_{2t} = i\kappa_{2t}$

$$\Gamma \simeq \frac{i\sqrt{\sin^2\theta_i - (n_2/n_1)^2} - (n_2/n_1)^2 \cos\theta_i}{i\sqrt{\sin^2\theta_i - (n_2/n_1)^2} + (n_2/n_1)^2 \cos\theta_i}$$



$$\frac{\phi}{2} = \cot^{-1}\left(\frac{\sqrt{\sin^2\theta_i - (n_2/n_1)^2}}{(n_2/n_1)^2 \cos\theta_i}\right) = \cot^{-1}\left(\frac{k_1^2}{k_2^2}\frac{\kappa_{2t}}{k_{1t}}\right)$$

$$\cot\left(\frac{k_{1t}a}{2} - \frac{m\pi}{2}\right) = \frac{k_1^2}{k_2^2}\frac{\kappa_{2t}a}{k_{1t}a}$$

m: even $\cot\left(\frac{k_{1t}a}{2}\right) = \frac{k_1^2}{k_2^2}\frac{\kappa_{2t}a}{k_{1t}a}$

m: odd $\tan\left(\frac{k_{1t}a}{2}\right) = -\frac{k_1^2}{k_2^2}\frac{\kappa_{2t}a}{k_{1t}a}$

TE mode

$$\Gamma \simeq \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{k_{1t} - i\kappa_{2t}}{k_{1t} + i\kappa_{2t}}$$

$$\Gamma \simeq \frac{n_1 \cos\theta_i - i n_2 \sqrt{\left(n_1/n_2\right)^2 \sin^2\theta_i - 1}}{n_1 \cos\theta_i + i n_2 \sqrt{\left(n_1/n_2\right)^2 \sin^2\theta_i - 1}} = \frac{\cos\theta_i - i\sqrt{\sin^2\theta_i - \left(n_2/n_1\right)^2}}{\cos\theta_i + i\sqrt{\sin^2\theta_i - \left(n_2/n_1\right)^2}}$$

$$\frac{\phi}{2} = \tan^{-1}\left(\frac{\sqrt{\sin^2\theta_i - \left(n_2/n_1\right)^2}}{\cos\theta_i}\right) = \tan^{-1}\left(\frac{\kappa_{2t}}{k_{1t}}\right)$$

$$\tan\left(\frac{k_{1t}a}{2} - \frac{m\pi}{2}\right) = \frac{\kappa_{2t}a}{k_{1t}a}$$

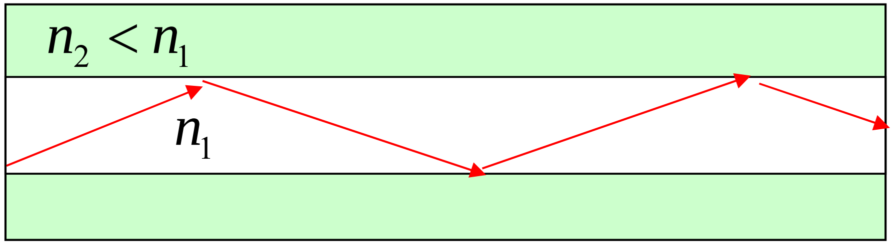
m: even $\tan\left(\frac{k_{1t}a}{2}\right) = \frac{\kappa_{2t}a}{k_{1t}a}$

For m=0, no cutoff

m: odd $\cot\left(\frac{k_{1t}a}{2}\right) = -\frac{\kappa_{2t}a}{k_{1t}a}$

Optical fibers

The optical fiber has cylindrical symmetry so the problem is better solved by using a cylindrical coordinate



Similar to the previous problem, the optical fiber always has a lowest propagating mode which has no cutoff frequency (or wavelength)

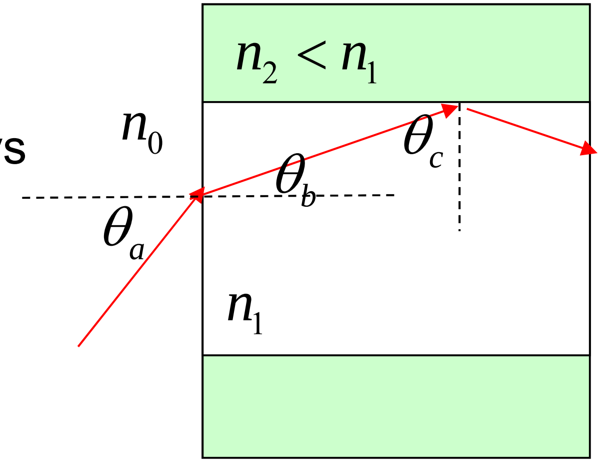
The next mode will occur when the wavelength obeys

$$\lambda < \frac{2\pi a\sqrt{n_1^2 - n_2^2}}{k_{01}}$$

In which k_{01} is the first root of the $J_0(x)$.

If the optical fiber can sustain only one mode, it is called a single mode fiber, otherwise, a multi-mode fiber

Numerical aperture



Consider a light being fed into the fiber from one end. The incident light and the refracted light obeys

$$n_0 \sin\theta_a = n_1 \sin\theta_b$$

Recall that the refracted light can only be a propagating light through the fiber if

$$\sin\theta_c = \cos\theta_b > \frac{n_2}{n_1}$$

$$\Rightarrow \left(\frac{n_0}{n_1}\right)^2 \sin^2\theta_a < 1 - \left(\frac{n_2}{n_1}\right)^2 \qquad \sin\theta_a < \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

The critical value is referred to as the numerical aperture $NA = \dfrac{\sqrt{n_1^2 - n_2^2}}{n_0}$

Resonant cavity

Add caps on the square waveguides of length *c*, one can make a resonant cavity

The solutions now should obey the boundary conditions

At $z=0, z=c$ $\quad E_x = E_y = 0$

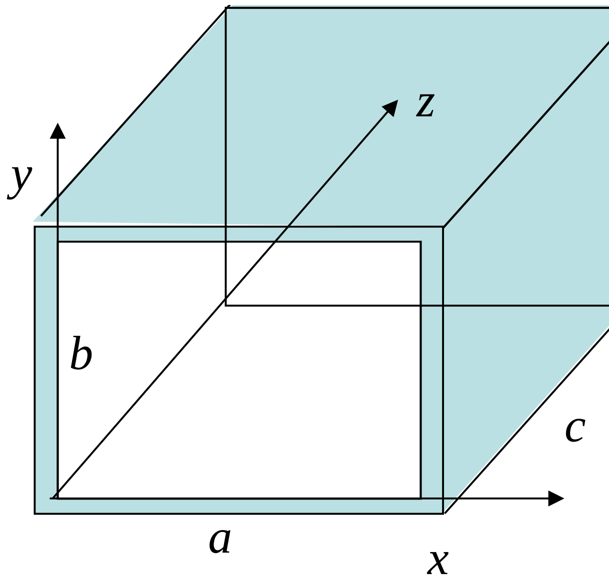
$$B_z = 0$$

$$E_x = \frac{i}{\left(\omega^2/c^2\right) - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right)$$

$$E_y = \frac{i}{\left(\omega^2/c^2\right) - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$

$$\Rightarrow \left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2}\right)E_x = \frac{\partial}{\partial z}\frac{\partial E_z}{\partial x} + i\omega\frac{\partial B_z}{\partial y}$$

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2}\right)E_y = \frac{\partial}{\partial z}\frac{\partial E_z}{\partial y} - i\omega\frac{\partial B_z}{\partial x}$$



TE mode

$$E_z = 0$$

$$\mathbf{B}(x, y, z, t) = \mathbf{B}_0(x, y)\sin(k_l z)e^{i\omega t}$$

$$k_l = \frac{\pi l}{c}$$

$$B_z = B_0 \cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\sin\left(\frac{l\pi z}{c}\right)e^{i\omega t}$$

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2}\right)E_x = i\omega\frac{\partial B_z}{\partial y}$$

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2}\right)E_y = -i\omega\frac{\partial B_z}{\partial x}$$

$$\frac{\omega}{c} = \pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{c}\right)^2}$$

Resonant mode

TM mode

$$B_z = 0$$

$$\mathbf{E}(x, y, z, t) = \mathbf{E}_0(x, y)\cos(k_l z)e^{i\omega t}$$

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2}\right)E_x = \frac{\partial}{\partial z}\frac{\partial E_z}{\partial x}$$

$$E_z = E_{z0}\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\cos\left(\frac{l\pi z}{c}\right)e^{i\omega t}$$

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2}\right)E_y = \frac{\partial}{\partial z}\frac{\partial E_z}{\partial y}$$

$$E_x = E_{x0}\cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\sin\left(\frac{l\pi z}{c}\right)e^{i\omega t}$$

$$E_y = E_{y0}\sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\sin\left(\frac{l\pi z}{c}\right)e^{i\omega t}$$

$$\frac{\omega}{c} = \pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{c}\right)^2}$$

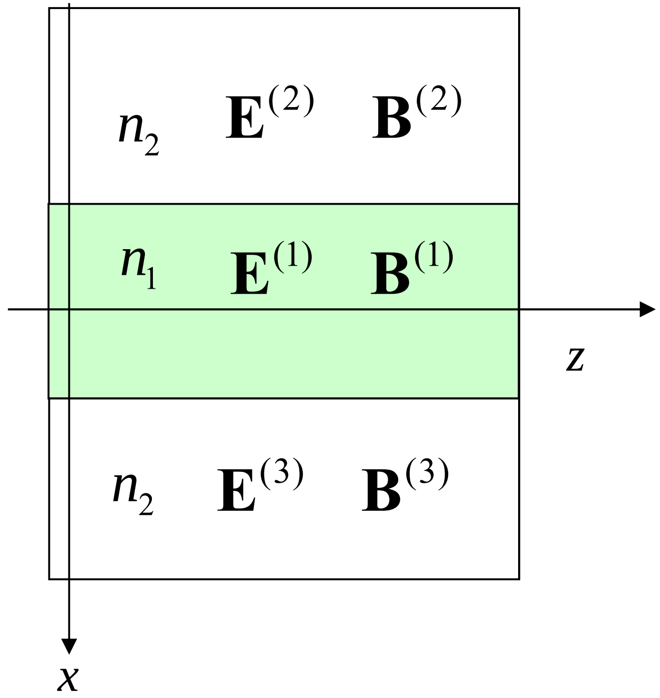
Homework

1. Solve the problem of a slab of dielectric by a general approach that assumes EM field in 3 regions:

For TE mode,

$$\frac{\partial^2 B_z^{(i)}}{\partial x^2} + \frac{\partial^2 B_z^{(i)}}{\partial y^2} + \left(\frac{\omega^2}{c^2} - k_i^2 \right) B_z^{(i)} = 0$$

$$k_1 = n_1 \frac{\omega}{c} \qquad k_2 = k_3 = n_2 \frac{\omega}{c}$$



There is a translational symmetry in y which yields

$$\frac{\partial^2 B_z^{(i)}}{\partial x^2} + k_{it}^2 B_z^{(i)} = 0$$

The propagation mode requires that $k_{2t}^2 = k_{3t}^2 < 0$

Write down the possible solutions to the differential equation in 3 regions and apply the following boundary conditions

$$x = -\frac{a}{2} \qquad H_z^{(1)} = H_z^{(2)} \qquad\qquad x = \frac{a}{2} \qquad H_z^{(1)} = H_z^{(3)}$$

Homework

2. The dielectric waveguides allow at least one mode of light to propagate, while the conductor waveguides do not. Why? Is this statement still true if the outer cladding thickness is not infinite?