Multi-order of High frequency Chebyshev band-pass filters center at 2.1414 GHz and bandwidth 642 MHz

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Abstract

Microwave transmission is a great invention on human history, for example, cell phone has already been indispensable in our lives, and high frequency of microwave engineering is especially important, so in this point of view, we construct high frequency of band-pass filters, center frequency at 2.1414GHz, and compare different orders of these. Results suggest that higher order of band-pass filter indeed has greater rate of decay, but rate of decay doesn't grow linearly when conducting higher order of filter, which suggest that the more we want, the greater payoff would come! Secondly, asymmetry of decay rate on both side of bandwidth is observed both on measurement results and simulation.

Keywords

Band-pass filters, microwave engineering, Chebyshev filters, high frequency filters

Introduction

Nowadays wireless communications has developed, and microwave signal to transmit information has already been part of our live. By using transmitter we shoot high-frequency wave and modulation it, and receiver receive signal then demodulation it. The quality of reception depends on the frequency range and sensitivity of filter. Since the frequency of PHS (personal handyphone system) is around 2GHz, we wanted to make a filter applied for 2GHz frequency. Considering techniques and back ground knowledge we have, we produce 2^{nd},3^{rd} ,and 4^{th} Chebyshev filters, and the filter's center frequency is 2.1414GHz with bandwidth of 642MHz.

Method and Material

1.Filter Transformations

Impedance and Frequency Scaling(Fig 1.)

(i)Impedance scaling

In the prototype design, the prototype is normalized designs having a source impedance of $R_S = 1\Omega$ and a cutoff frequency of $\omega_c = 1$. A source resistance of R_0 can be obtained by multiplying the impedances of the prototype design by R_0.

Then, if we let primes denote impedance scaled quantities, we have the new filter component values given by

$$L' = R_0 \cdot L \qquad \text{(eq.1a)}$$

$$C' = \frac{C}{R_0} \qquad \text{(eq.1b)}$$

$$R'_S = R_0 \qquad \text{(eq.1c)}$$

$$R'_L = R_0 \cdot R_L \qquad \text{(eq.1d)}$$

Where L, C, are the component values for the original prototype.

(ii)Frequency scaling for low-pass filters.

To change the cutoff frequency of a low-pass prototype from unity to ω_c requires that we scale the frequency dependence of the filter by the factor $1/\omega_c$, which is accomplished by replacing ω by ω/ω_c:

$$\omega \leftarrow \frac{\omega}{\omega_c} \qquad \text{(eq.2)}$$

where ω_c is the new cutoff frequency.

The new element values are determined by applying the substitution of (eq.2) to the series reactances , $j\omega L_k$, and shunt susceptances, $j\omega C_k$,of the prototype filter.

Thus

$$jX_k = j\frac{\omega}{\omega_c}L_k = j\omega L'_k \ ,$$

$$jB_k = j\frac{\omega}{\omega_c}C_k = j\omega C'_k \ ,$$

which shows that the new element values are given by

$$L'_k = \frac{L_k}{\omega_c} \qquad \text{(eq.3a)}$$

$$C'_k = \frac{C_k}{\omega_c} \qquad \text{(eq.3b)}$$

When both impedance and frequency scaling are required, the results of (eq.1) can be combined with (eq.3) to give

$$L'_k = \frac{R_0 L_k}{\omega_c},$$

$$C_k' = \frac{C_k}{R_0 \omega_c}.$$

(iii)Bandpass transformations

If ω_1 and ω_2 denote the edges of the passband, then a bandpass response can be obtained using the following frequency substitution:

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \quad \text{(eq.4)}$$

where $\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$

Then the new filter elements are determined by using (eq.4) in the expressions for the series reactance and shunt susceptances. Thus,

$$jX_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) L_k = j\omega L_k' - j\frac{1}{\omega C_k'}$$

Which shows that a series inductor, L_k ,is transformed to a series LC circuit with element values,

$$L_k' = \frac{L_k}{\Delta \omega_0},$$

$$C_k' = \frac{\Delta}{\omega_0 L_k}.$$

Similarly,

$$jB_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) C_k = j\omega C_k' - j\frac{1}{\omega L_k'}$$

which shows that a shunt capacitor, C_k' , is transformed to a shunt LC circuit with element values, (Figure.2)

$$L_k' = \frac{\Delta}{\omega_0 C_k},$$

$$C_k' = \frac{C_k}{\Delta \omega_0}.$$

2.Bandpass Filters Using Capacitively Coupled Series Resonators

The type of bandpass filter that can be conveniently fabricated in microstrip form is the capacitive-gap coupled resonator filter shown in Fig 3.An 2th order filter of this form will use 2 resonator series sections of transmission line with 3 capacitive gaps between them. The filter can then be modeled as shown in Fig 4. The resonators are approximatelyλ/2 long at the center frequency,ω_0.

Secondly, we redraw the equivalent circuit of Fig 4. with negative-length transmission line sections on either side of the series capacitors. The lines of lengthΦ will beλ/2 long at ω_0, so the electrical length, θ_i,of the i-th section in Figure.3,4 is with ϕ_i<0.

$$\theta_i = \pi + \frac{1}{2}\emptyset_i + \frac{1}{2}\emptyset_{i+1} \qquad \text{(eq.5)}$$

The reason for doing this is that the combination of series capacitor and negative-length transmission line forms the equivalent circuit of an admittance inverter. In order for this equivalence to be valid, the following relationship must hold between the electrical length of the lines and the capacitive susceptance:

$$\phi_i = -\tan^{-1}(2Z_0B_i) \qquad \text{(eq.6a)}$$

Then the resulting inverter constant can be related to the capacitive susceptance as

$$B_i = \frac{J_i}{1-(Z_0J_i)^2} \qquad \text{(eq.6b)}$$

Between any two consecutive inverters we have a transmission line section that is effectively 2θ in length. This line is approximately $\lambda/2$ long in the vicinity of the bandpass region of the filter, and has an approximate equivalent circuit that consists of a shunt parallel LC resonator, as in Fig 5.

If we let $\omega = \omega_0 + \Delta\omega$, the shunt impedance Z_{12} can be written for small $\Delta\omega$ as

$$Z_{12} \cong \frac{-jZ_0\omega_0}{\pi(\omega-\omega_0)} \qquad \text{(eq.7)}$$

We know the impedance near resonance of a parallel LC circuit is

$$Z = \frac{-jL\omega_0^2}{2(\omega-\omega_0)} \qquad \text{(eq.8)}$$

with $\omega_0^2 = 1/LC$. Equation this to (eq.7) gives the equivalent inductor and capacitor values as

$$L = \frac{2Z_0}{\pi\omega_0} \qquad \text{(eq.9a)}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{\pi}{2Z_0\omega_0} \qquad \text{(eq.9b)}$$

The end sections of the circuit of Figure.6 require a different treatment. The lines of lengthθon either end of the filter are matched to Z_0,and so can be ignored. The end inverters, $J_1$and J_3, can each be represented as a transformer followed by a $\lambda/4$ section of line, as shown in Figure .9.The ABCD matrix of a transformer with a turns ratio 2 in cascade with a quarter-wave line is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & -jZ_0 \\ -jZ_0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-jZ_0}{2} \\ \frac{-jZ_0}{2} & 0 \end{bmatrix}$$

Comparing this to the ABCD matrix of an admittance inverter shows that the necessary turns ratio is $2 = JZ_0$. The $\lambda/4$ line merely produces a phase shift, and so can be ignored.

Using these results for the interior and end sections allow the circuit of Figure.6 to be transformed into the circuit of Fig 7.Next,we show that the admittance inverters have the effect of transforming a shunt LC resonator into a series LC resonator, leading to the final equivalent circuit of Fig 8. This will then allow the admittance inverter constants, J_n,to be determined from the element values of a low-pass prototype.

With reference to Fig 7., the admittance just to the right of the J_2 inverter is

$$j\omega C_2 + \frac{1}{j\omega L_2} + Z_0 J_3^2 = j\sqrt{\frac{C_2}{L_2}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) + Z_0 J_3^2$$

since the transformer scales the load admittance by the square of the turns ratio. Then the admittance seen at the input of the filter is

$$Y = \frac{1}{J_1^2 Z_0^2}\left\{j\omega C_1 + \frac{1}{j\omega L_1} + \frac{J_2^2}{j\sqrt{C_2/L_2}\left[\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] + Z_0 J_3^2}\right\}$$

$$Y = \frac{1}{J_1^2 Z_0^2}\left\{j\sqrt{C_1/L_1}\left[\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] + \frac{J_2^2}{j\sqrt{C_2/L_2}\left[\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] + Z_0 J_3^2}\right\} \qquad \text{(eq.10)}$$

These results also use the fact, from (eq.9),that $L_n C_n = 1/\omega_0^2$ for all LC resonators.
Now the admittance seen looking into the circuit of Fig 8. is

$$Y = j\omega C_1' + \frac{1}{j\omega L_1'} + \frac{1}{j\omega L_2' + 1/j\omega C_2' + Z_0}$$

$$Y = j\sqrt{\frac{C_1'}{L_1'}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) + \frac{1}{j\sqrt{L_2'/C_2'}\left[\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] + Z_0} \qquad \text{(eq.11)}$$

which is identical in form to (eq.10). Thus, the two circuits will be equivalent if the following conditions are met:

$$\frac{1}{J_1^2 Z_0^2}\sqrt{\frac{C_1}{L_1}} = \sqrt{\frac{C_1'}{L_1'}} \qquad \text{(eq.12a)}$$

$$\frac{J_1^2 Z_0^2}{J_2^2}\sqrt{\frac{C_2}{L_2}} = \sqrt{\frac{C_2'}{L_2'}} \qquad \text{(eq.12b)}$$

$$\frac{J_1^2 Z_0^2 J_3^2}{J_2^2} = Z_0 \qquad \text{(eq.12c)}$$

We know L_n and C_n from (eq.9); L_n' and C_n' are determined from the element values of a lumped-element low pass prototype which has been impedance scaled and frequency transformed to a bandpass filter. Using the result in Fig 2. and the impedance scaling formulas of (eq.1)allows the L_n' and C_n' values to be written as

$$L_1' = \frac{\Delta Z_0}{\omega_0 g_1} \qquad \text{(eq.13a)}$$

$$C_1' = \frac{g_1}{\Delta \omega_0 Z_0} \qquad \text{(eq.13b)}$$

$$L_2' = \frac{g_2 Z_0}{\Delta \omega_0} \qquad \text{(eq.13c)}$$

$$C_2' = \frac{\Delta}{\omega_0 g_2 Z_0} \qquad \text{(eq.13d)}$$

where $\Delta = \frac{(\omega_2 - \omega_1)}{\omega_0}$ is the fractional bandwidth of the filter. Then (eq.12)can be solved for the inverter constants with the following results:

$$J_1 = \frac{1}{Z_0}\left(\frac{C_1 L_1'}{L_1 C_1'}\right)^{1/4} = \frac{1}{Z_0}\sqrt{\frac{\pi\Delta}{2g_1}} \qquad \text{(eq.14a)}$$

$$J_2 = J_1 Z_0\left(\frac{C_2 C_2'}{L_2 L_2'}\right)^{1/4} = \frac{1}{Z_0}\frac{\pi\Delta}{2\sqrt{g_1 g_2}} \qquad \text{(eq.14b)}$$

$$J_3 = \frac{1}{Z_0}\frac{J_2}{J_1} = \frac{1}{Z_0}\sqrt{\frac{\pi\Delta}{2g_2}} \qquad \text{(eq.14c)}$$

The design equations for a bandpass filter with Nth order capacitive-gap coupled resonator filter are

$$J_1 = \frac{1}{Z_0}\sqrt{\frac{\pi\Delta}{2g_1}}$$

$$J_n = \frac{1}{Z_0}\frac{\pi\Delta}{2\sqrt{g_{n-1}g_n}} \qquad \text{for n=2,3,...,N,}$$

$$J_{n+1} = \frac{1}{Z_0}\sqrt{\frac{\pi\Delta}{2g_N g_{N+1}}}$$

We use (eq.14) to find the admittance inverter constants,J_i,from the low-pass prototype value (g_i) and the fractional bandwidth,Δ.There will be N+1 inverter constants for Nth order filter. Then (eq.6b) can be used to find the susceptance,B_i,for the i-th coupling gap.

$$B_i = \frac{J_i}{1-(Z_0 J_i)^2} \qquad \text{(eq.6b)}$$

Finally,the electrical length of the resonator sections can be found from (eq.6a) and (eq.5):

$$\theta_i = \pi - \frac{1}{2}[\tan^{-1}(2Z_0B_i) + \tan^{-1}(2Z_0B_{i+1})]$$

The center of frequency is 2.1414GHz, which would cause the width between the connecting transmission lines be too small, over our construction techniques, so we replaced this by using capacitor, and the magnitude of capacitor is determined by the following formula.

$$C_i = \frac{B_i}{\omega_0}$$

Chebyshev filter	2nd	3rd	4th
f_0(GHz)	2.1414		
f_1(GHz)	1.84415		
f_2(GHz)	2.48657		
c_1(pf) real/ approximate	1.978/2	1.001/1	2.179/2
c_2(pf) real/ approximate	0.5724/0.5	0.5000/0.5	0.5008/0.5
c_3(pf) real/ approximate	0.9637/1	0.5000/0.5	0.4084/0.4
c_4(pf) real/ approximate		1.001/1	0.5008/0.5
c_5(pf) real/ approximate			1.017/1
d_1(m)	0.0265	0.0285	0.02662
d_1(m)	0.0283	0.0306	0.03104
d_1(m)		0.0285	0.03104
d_1(m)			0.02843

Table 1. Magnitude of capacitors and length of transmission lines for construction.

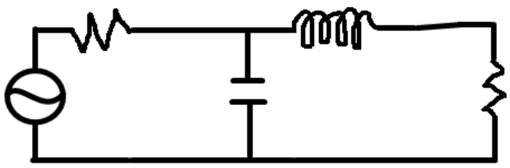

Figure 1. Low-pass filter prototype,N=2.

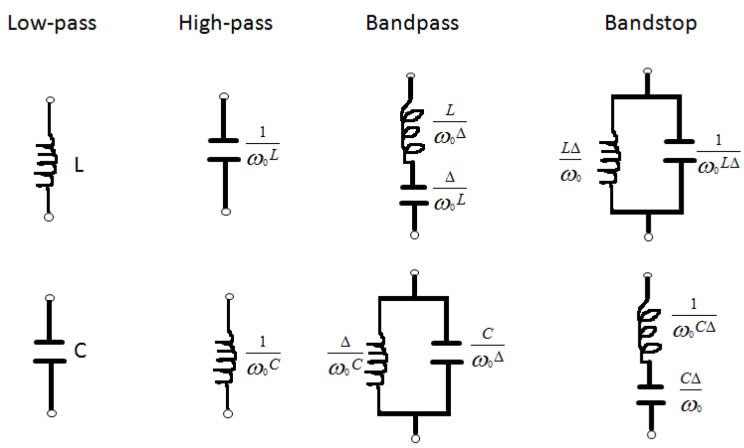

Figure 2. Summary of Prototype Filter Transformations.

Figure 3. The capacitive-gap coupled resonator bandpass filter.

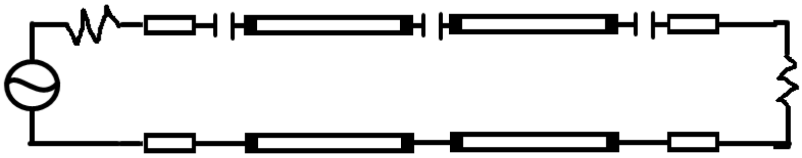

Figure 4. Transmittion line model.

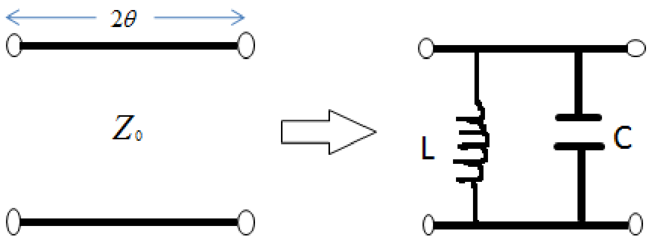

Figure 5. Equivalent circuit of the transmission line of length 2θ.

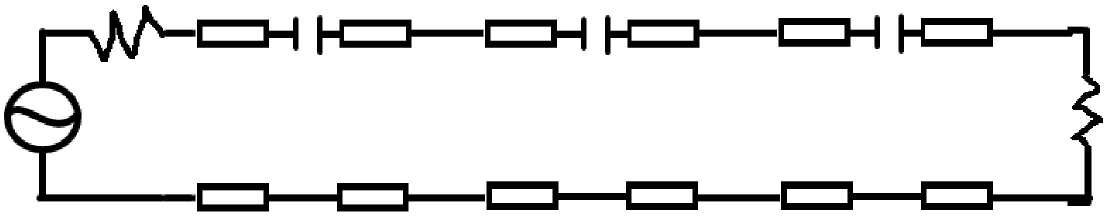

Figure 6. Transmission line model with negative-lengh sections forming admittance inverters.

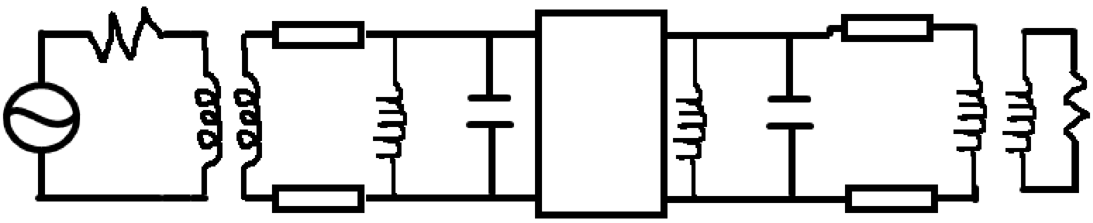

Figure 7. Using results of Figure.5 and Figure.9 for the N=2 case.

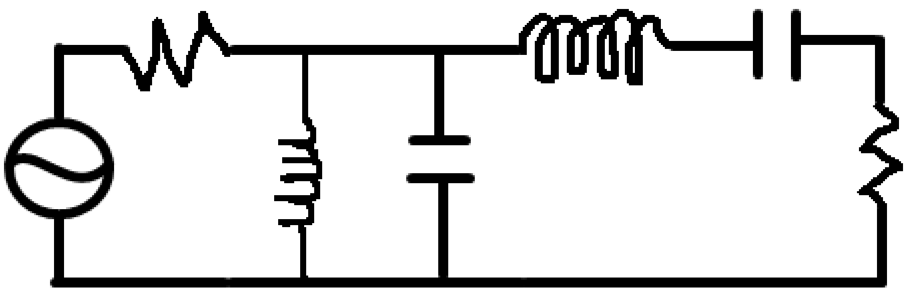

Figure 8. Lumped-element circuit for a bandpass filter for N=2.

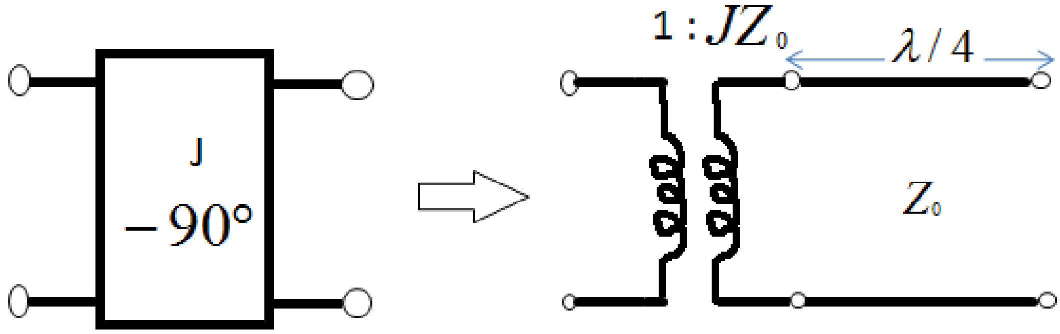

Figure 9. Equivalent circuit of the admittance inverters.

Results and discussion

We design three different levels of Chebyshev band-pass filters, second level, third level, and fourth level and compare S-parameter both by measurements and simulation. The simulation model is based on Chebyshev Response formula. Ripple is denoted as R, Chebyshev polynomial denoted as $C_n(x)$, and then the formula is $|S_{21}(x)|^2 = \dfrac{1}{1+\varepsilon^2 C_n^2(x)}$ $\varepsilon = \sqrt{e^{R/10} - 1}$

However this formula is suited for low-pass filter, from low-pass to band-pass, one may play a trick on this formula just by a change of variable. $x = \dfrac{1}{\Delta}\left[\dfrac{f}{f_0} - \dfrac{f_0}{f}\right]$,

f_0 is the band center, $\Delta = \dfrac{BW}{f_0}$, BW is the bandwidth.

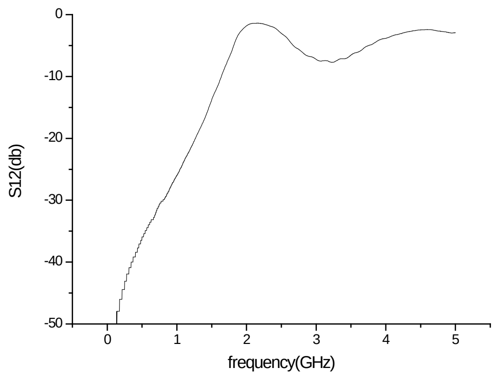

Figure10. S12 measurement of Chebyshev 2nd band-pass filter.

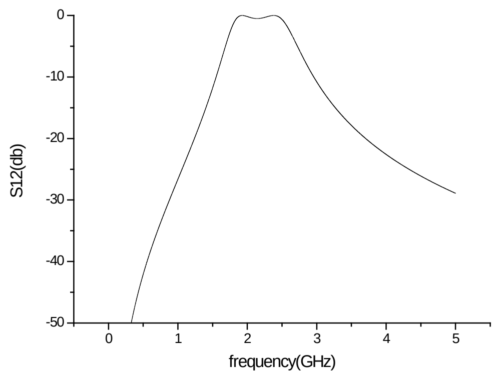

Figure11. simulation result of Chebyshev 2nd filter.

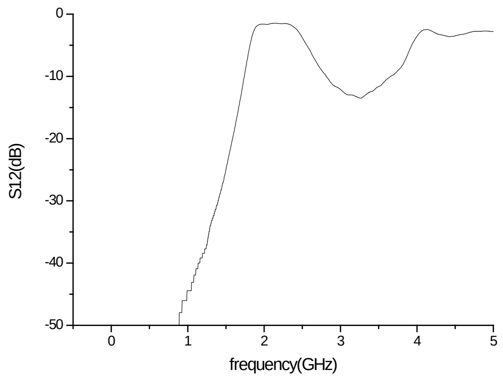

Figure12. S12 measurement of Chebyshev 3rd band-pass filter.

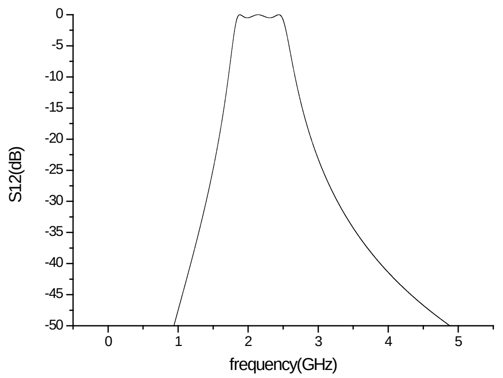

Figure13. simulation result of Chebyshev 3rd filter.

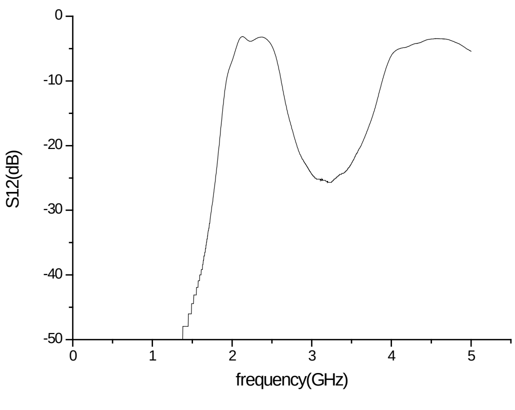

Figure 14. S12 measurement of Chebyshev 4th band-pass filter.

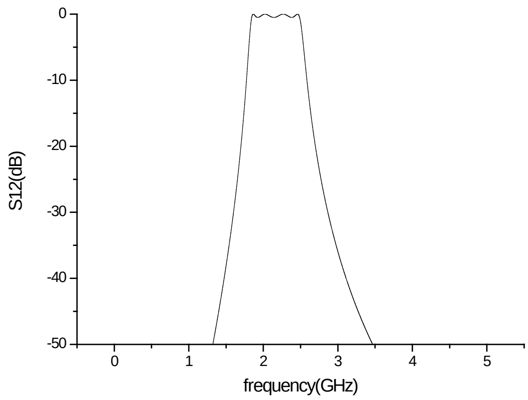

Figure 15. simulation result of Chebyshev 4th filter.

One may have discovered that the higher order of the band-pass filter, the greater rate of decay. For 2nd Chebyshev band-pass filter, the band center f_0 at 2.1414(GHz), from 1.320 GHz to 1.844 GHz, the filter could be viewed decaying linearly (R-square>0.9), so we fit this range and the slope would represent the rate of decay. For 2nd Chebyshev filter, the rate of decay is 27.92dB/GHz, and 34.57dB/GHz for simulation. For 3rd Chebyshev filter, rate of decay is 58.54dB/GHz, 65.43dB/GHz for simulation. For 4th Chebyshev filter, rate of decay is 57.86 dB/GHz, 93.24dB/GHz for simulation. You may be confused of this results, because 3rd filter shows greater rate of decay than 4th filter, but if you view carefully on Fig 12. and Fig 14., 4th filter rate of decay is obviously greater than 3rd by eye-ball, so what happen here is 4th filter shift a little bit to the right, from f_0=2.1414 GHz to f_0=2.2414GHz, therefore we may alter the range of fitting, and the rate of decay turns out to be 76.45dB/GHz after adjustment.

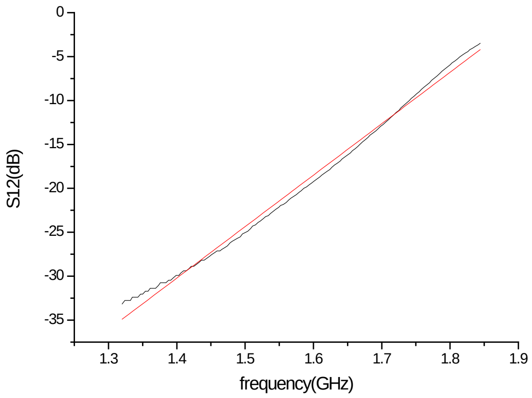

Figure 16. Linear fit of 3rd Chebyshev filter, from 1.320GHz to 1.844GHz, slope=58.57 dB/GHz, R-square=0.994

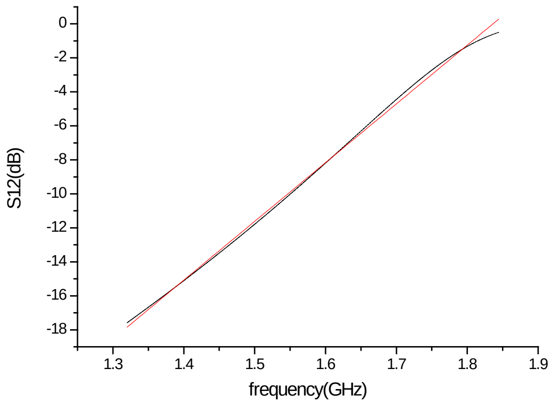

Figure 18. Linear fit of 2nd Chebyshev filter simulation, from 1.320GHz to 1.844GHz, slope=34.57 dB/GHz, R-square=0.999

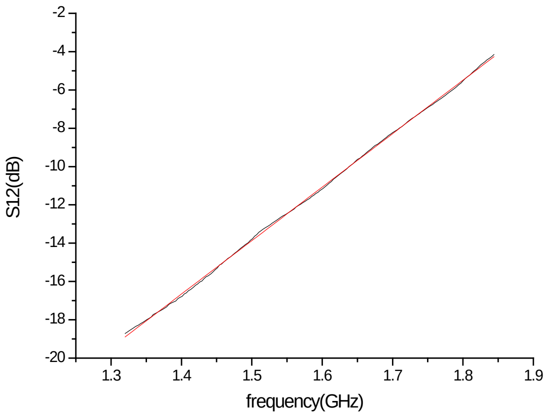

Figure 17. Linear fit of 2nd Chebyshev filter, from 1.320GHz to 1.844GHz, slope=27.92 dB/GHz, R-square=0.999

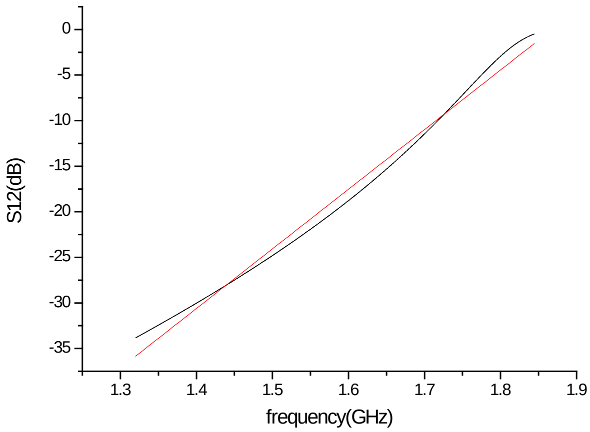

Figure 19. Linear fit of 3rd Chebyshev filter simulation, from 1.320GHz to 1.844GHz, slope=65.43 dB/GHz, R-square=0.99

	2^{nd} Chebyshev filter	3^{rd} Chebyshev filter	4^{th} Chebyshev filter
measurement	27.92	58.57	76.45
Simulation	34.57	65.43	93.24

Table 2. Rate of decay(dB/GHz) for real and simulation results

Higher level of band-pass filter indeed has greater rate of decay. The rate of decay of 3^{rd} Chebyshev filter is 210% of 2^{nd} Chebyshev filter, however the rate of decay of 4^{th} Chebyshev filter is 130% of 3^{rd} Chebyshev filter, which suggest that the greater rate of decay we want to achieve, the payoff would be greater. In other words rate of decay doesn't grow linearly while we conducting higher level of band-pass filter. Though we didn't analyze the rate of decay on right side of bandwidth, but one could observe right side of bandwidth decay rate is less than left side by eye-ball, and this phenomena could be observed on simulation results also, but for measurement results, right side of bandwidth rate of decay is far cry from simulation results, and we still haven't found out a reasonable explanation, however, this may be an effect of we used approximate magnitude of capacitor for the actual value of capacitor.

Conclusion

Asymmetry of decay rate on both side of bandwidth is observed both on measurement results and simulation. Higher level of band-pass filter indeed has greater rate of decay, but rate of decay doesn't grow linearly when conducting higher level of filter, which suggest that the more we want, the greater payoff would come!

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